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Managing risks from climate impacted hazards – The value of investment flexibility under uncertainty☆

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ABSTRACT

Incomplete knowledge about climate change and the related uncertainty in climate prediction makes adaptation inherently difficult. We introduce a real options framework to determine the optimal adaptation to catastrophic risk that takes into account climate change uncertainty. The framework can be used to select the optimal adaptation project from a number of alternative projects or to determine the optimal investment sequence of the considered projects. In applying the model to the management of bushfire risk at a local government area, we find that the framework can significantly increase the value of adaptation investment, above the current net present value, and also improve upon deterministic dynamic models. We also find that it is important to consider sequential investment to preserve investment flexibility under the uncertainty of climate change. When decision makers can afford multiple investment projects, the loss associated with the use of a simple net present value rule can be substantial, and it is important to use a deterministic dynamic model or a real options model.

1. Introduction

Climate induced catastrophes including floods, storm surges and bushfires, are predicted to occur more frequently and cause more severe damages in future periods due to the impact of climate change. With a higher global temperature, the climate system is more energetic, and catastrophes become more likely to occur (Solomon, 2007). The increased frequency and severity of natural disasters in recent decades have made such a future outlook particularly concerning and serious attention has been paid to climate change adaptation that mitigates catastrophic risks (Van Aalst, 2006).

Optimal adaptation to climate change is challenging, posing several difficult issues. First, a risk quantification framework is required that can incorporate increasing losses induced by economic development in the region as well as the possible growth of loss frequency and severity due to climatic change. Second, the framework needs to quantify the level of uncertainty related to climate change effects in order to incorporate the impact of uncertainty into the investment decision. With respect to this second issue, the usual approach of using historical climate data to estimate climate change uncertainty may not be satisfactory. Changes in the climate system typically occur slowly, with significant lags from the time when emissions are released into the atmosphere. Therefore using historical climate data to predict future climate change may result in conservative estimates that may not reflect the impact of new and recent emissions or feedback mechanisms. Third, with uncertainty about the extent to which the climate will change, and the irreversibility of investment projects, the opportunity to invest in an adaptation project is analogous to a financial call option, and optimal investment needs to take into account the investment option value. In addition, there are usually several adaptation projects that could be selected. Therefore, issues related to the optimal selection of investment projects as well as the optimal sequence of adaptation investment, i.e., which projects to be invested first and which ones to invest later, need to be dealt with.

The problem of evaluating catastrophic loss reduction investment under the impacts of climate change has been examined in previous studies. West, Small, and Dowlatabadi (2001) provide a framework to evaluate increased damage from storm surge when the sea level rises over time. They use the loss distribution approach (LDA) to model house losses and insurance premium data to estimate the parameters of the model. Brouwer and van Ek (2004) examine flood risk management for an area in the Netherlands where ecological benefits are also important. Michael (2007) evaluates the damage of storm surge in Maryland

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under climatic change, using the reduction in house elevation under increased sea levels to determine the additional costs of insurance attributed to sea level rise. Zhu, Lund, Jenkins, Marques, and Ritzema (2007) examine time varying flood risk management strategies to maximize the total reduction in the expected loss for a floodplain in California.

Kirshen, Knee, and Ruth (2008) study the increased damage caused by storm surge in Boston when the sea level rises, and the region responds by adapting to the heightened risk. They construct various scenarios that differ in the number of storm surges, the extent of sea level rise and the level of adaptation, and then evaluate the expected damage in each scenario. Bouwer, Bubeck, and Aerts (2010) examine the potential risk posed by river flooding in the Netherlands. In their model, increases in the frequency of flood events under climate change are estimated using data on 10 days accumulative rainfall. They estimate flood damage by using an engineering model that specifies the relationship between the flood damage and flood depth. Mathew, Trück, and Henderson-Sellers (2012) examine adaptation options that help to reduce flood risk in India. They estimate loss frequency and severity distributions using a Bayesian framework. Their proposed framework helps to combine information provided by experts with observed data on flood events in a statistically consistent way. They rank adaptation measures based on the net present value (NPV) rule, and a triple bottom line that considers environmental benefits. Tsvetanov and Shah (2013) examine the problem of optimal investment timing for protection measures against storm surges in Connecticut. For each time period, they estimate a risk curve that relates flood damage to different return periods using the HAZUS-MH MR4 risk assessment model supplied by the Federal Emergency Management Agency, and then derive the expected loss using the estimated curves. The optimal time of adaptation is the time when the expected NPV of the investment project reaches the maximum level. Truong and Trück (2016a) examine the impact of risk aversion and optimal investment timing on the value of an investment project that reduces bushfire risk in an urban area. They extend the standard LDA to allow for the impacts of increased loss due to economic growth and higher frequency of bushfires as a result of climate change.

In most previous studies a simple NPV rule is used to determine the investment decision, i.e., a project is invested if its NPV is positive. However, the NPV rule does not consider the possibility that the project can be invested at some time in the future. Exceptions are the studies by Tsvetanov and Shah (2013); West et al. (2001); Zhu et al. (2007) and Truong and Trück (2016a) who depart from applying a standard NPV rule and examine the optimal timing of adaptation investments. The authors determine the optimal moment to invest by maximizing the NPV of a project over all possible investment times. However, none of these studies considers the impact of climate change uncertainty on investment decisions and the value of investment flexibility that helps to cope with the uncertainty of climate change. In this paper, we consider the uncertainty of catastrophe frequency that is increasing in the forecast horizon.

Flexibility in decision making has been widely recognised to play an important role in adaptation under climate change uncertainty. In recent years, policy makers have suggested to follow adaptation plans that 'maximize flexibility, keep options open and avoid lock-in' (Kuijken, 2010). Several studies have followed the robust adaptation approach to achieve such objectives. For example, Haasnoot, Kwakkel, Walker, and Ter Maat (2013) consider the uncertainty about future climate and socio-economic developments when examining dynamic adaptive water management pathways for the lower Rhine Delta in the Netherlands. Lempert and Groves (2010) examine robust adaptive water management plans for Southern California, considering uncertainty for a range of factors including future climate, future water demand, groundwater recharge, annual cost, and the effectiveness of management strategies. These approaches, however, are not without criticisms (Mills et al., 2014; Truong & Trück, 2016b). By maximizing flexibility regardless of the opportunity cost, the benefit of durable investment and long lasting policies will be foregone. As suggested by Mills et al. (2014), the price of maximizing flexibility at any cost can be quite high. Our paper provides a method to consider adaptation pathways – which in the following we will refer to as investment sequences – with the important feature that we take into account both the benefit and the cost of flexibility.

In this paper, we extend the framework by Truong and Trück (2016a) in three important directions. First, we incorporate the impact of climate change uncertainty into the investment decisions. In contrast, Truong and Trück (2016a) assume that the impact of climate change is deterministic and completely known at the initial time when investment decisions are examined. Second, we consider the investment decision when several competing projects are available, while previous studies typically only investigate a single investment project. Third, we also analyze optimal investment sequences or so-called 'investment pathways'. This consideration is important since it provides important insights into the value of flexibility under the uncertainty of climatic change. Investment sequences are not considered by Truong and Trück (2016a) since they study the case of a single investment.

To model the impact of climate change uncertainty on the occurrence of loss events, we use a doubly stochastic Poisson process that has also been proposed for the pricing of catastrophic bonds, see, e.g., Lin, Chang, and Powers (2009) and widely used in the credit risk literature (Lando, 2009). In analysing investment decisions, we construct a real options framework that allows for the selection of an investment project from several alternatives as well as for the management of sequential investment into different projects. Although the problem of optimally selecting a project from many alternatives has been examined by (Décamps, Mariotti and Villeneuve, 2006), their discussion is quite technical and may be inaccessible to many.

Our proposed continuous-time model allows for closed-form solutions and is relatively easy to implement. In a case study that examines the problem of bushfire risk management for a local area, we illustrate that the consideration of investment flexibility can significantly increase the value of adaptation investment above the current NPV. In addition, the investment value can be further increased by considering the optimal sequence of projects to be invested. In contrast, ignoring investment options and investment sequencing can result in the elimination of adaptation options that are valuable for the management of future risk. A deterministic model can help to reduce the loss significantly, but the investment value can be further improved by the application of a real options model.

Furthermore, the results on managing sequential investment suggest that it is optimal to invest in a low sunk cost project first and preserve the flexibility of investing in a high sunk cost project for the possible case when the catastrophic risk is higher. This result is also consistent with findings provided by earlier studies that are more focused on the qualitative analysis of adaptation strategies, see, e.g., Hallegatte (2009). Conducted sensitivity tests suggest that the loss due to the use of a simple NPV rule is highly sensitive to investment cost and the expected loss severity, while the loss due to the use of a deterministic dynamic model is highly sensitive to uncertainty, the discount rate and the expected severity. This implies that for large scale projects, it is important to at least base the investment decision on a deterministic dynamic model. When both uncertainty and the scale of the project are large, it is important to use a real options model.

The remainder of the paper is structured as follows. In Section 2, the modeling framework is outlined and analyzed. In

Section 3, the proposed framework is applied in a case study of bushfire risk management in a Sydney local government area. The section also analyses the investment decision for the case where only one project can be selected from several alternatives, as well as the case where investment sequences are considered. Section 4 concludes and discusses the implications as well as the limitations of the proposed framework.

2. Modeling framework

In the following, we provide a framework for modeling catastrophic risks and the quantification of potential losses from extreme events. Our approach takes into account the growth of loss severity and the uncertain development of the frequency of events under climate change. We then analyze investment models to select one project from several alternatives and to determine the optimal sequence of projects when several projects can be invested.

2.1. Catastrophic risk modelling

Similar to Truong and Trück (2016a), we extend the LDA to model catastrophic risk. The LDA assumes that the total loss over a period $(0, t]$ is modeled as a compound Poisson process:

$$S_t = \sum_{n=1}^{N_t} X_n, \tag{2.1}$$

where N_t denotes the number of catastrophic events occurring from time 0 up to time t, and X_n is the loss caused by the nth event. In this standard model, N_t is assumed to follow a homogeneous Poisson process with intensity $\Lambda > 0$, X_n is assumed to be independently and identically distributed according to a distribution $H(X)$ and X_n is independent from N_t. A realization of two catastrophic events with severities x_1, x_2 over period $(0, t]$ corresponds to $\{N_t = 2; X_1 = x_1, X_2 = x_2\}$. For a discussion of the application of the LDA in different areas, see Truong and Trück (2016a).

We extend the standard model (2.1) to allow for growing loss severity and frequency as follows. To allow the loss severity to grow over time, we follow Truong and Trück (2016a) to model the catastrophic loss X_n as a product of the catastrophic loss under zero growth X_0 and a growth component:

$$X_n = X_0 e^{\gamma \tau_n}. \tag{2.2}$$

In Eq. (2.2), γ is the growth rate of loss exposure, and τ_n is the random time when the nth climate impacted event occurs, which is determined by a Poisson process. Loss exposure may grow over time due to the increases in the number of properties in an area or due to additional investment in infrastructure. In addition, as the economy grows, residents tend to improve their properties and assets.[1] X_0 is assumed to be identically, independently distributed and X_0 is independent from N_t and, therefore, independent from τ_n. In the following, we denote the expected value of X_0 by β.

Climate change is modelled to increase the frequency of catastrophic events, as suggested by previous studies, e.g., Hasson, Mills, Timbal, and Walsh (2009); Truong and Trück (2016a). Different from Truong and Trück (2016a) who assume that the extent of climate change is known, we allow climate change to be uncertain in our model. Previous adaptation studies, such as Fisher and Rubio (1997); Gersonius, Ashley, Pathirana, and Zevenbergen (2013), have used Geometric Brownian Motion (GBM) processes to model uncertainty related to climate variables. As a result of the stochastic variation in these variables, the frequency of catastrophic events will also vary stochastically. As such, we assume that the number of catastrophic events N_t that occur over period $(0, t]$ follows a doubly stochastic Poisson process, with the intensity Λ_t of the process evolving according to a Geometric Brownian Motion (GBM):

$$d\Lambda_t / \Lambda_t = \mu dt + \sigma dW_t, \tag{2.3}$$

where W_t is a Wiener process, μ is the expected growth rate of Λ_t and σ represents the magnitude of the uncertainty in predicting future values of Λ_t.

2.2. Optimal investment for an individual project

In this section, we examine the optimal investment decision for a project that helps to reduce the frequency of catastrophic events by a proportion k. The investment cost of the project is I and the maintenance cost flow is C. The project is assumed to last infinitely and the investment cost is sunk once committed, following the typical assumptions adopted in other real options studies (Baranzini, Chesney, & Morisset, 2003; Dixit & Pindyck, 1994; Fisher, 2000; Gollier & Treich, 2003; Pindyck, 2002). In empirical applications, a project with an infinite lifetime is replicated using consecutive finite lifetime projects.

To determine the optimal investment decision, we need to know the value of the project and the value of the option to invest. Although the investment option is not traded in the market, in many aspects, it is similar to new products in financial markets and the pricing approach used for pricing new financial products can be applied. In financial markets, investors require premiums for bearing (systematic) risk, such that assets with higher (systematic) risk are typically assumed to have lower prices and higher returns. Not all risk requires a premium though: non-systematic risk that is uncorrelated with the market and can therefore be diversified in large portfolios does not affect asset prices.

After deducting the risk premium from asset returns, all assets provide a risk free rate of return. This important insight leads to the risk neutral pricing approach: we create a risk neutral probability measure that is equivalent to the physical probability measure by setting the rate of return of a risky asset with a known price under the risk neutral measure to equal the risk free rate of return. Then the risk neutral probability measure can be used to price new assets. The price of any random cash flow can be obtained by taking the discounted expected value of the cash flow under the risk neutral probability measure, where the discount rate is equal to the risk free rate. The risk neutral measure makes asset pricing much easier, since pricing now involves only the expected value, not variance or higher moments. Studies by Harrison and Kreps (1979) and Harrison and Pliska (1983) also show that the existence of a risk neutral probability measure is equivalent to no arbitrage.

In this paper, we assume that catastrophic events are uncorrelated with financial market returns, and catastrophic risk is a diversifiable risk. As a result of these assumptions, no risk premium is required for bearing catastrophic risk and as formally shown by Jarrow, Lando, and Yu (2005), under the risk neutral probability measure, the Poisson intensity process has the same drift and volatility as under the physical probability measure. Unchanged Poisson intensity processes under the risk neutral measure have also been used in weather derivatives pricing studies, see e.g., Chang, Yang, and Yu (2016); Turvey (2005); Xu, Odening, and Musshoff (2008).

Under the risk neutral measure, the insurance premium for insuring losses over period $(0, t]$ is the discounted expected loss (Jang & Krvavych, 2004):

$$p_{0,t} = E\left[\sum_{n=1}^{N_t} e^{-r\tau_n} X_n | \mathscr{F}_0\right] \tag{2.4}$$

[1] Previous studies, see, e.g., Brouwer and van Ek (2004); Crompton, McAneney, and Leigh (2006); Crompton and McAneney (2008), also suggest that natural disaster losses grow exponentially over time.

where r is the risk free rate that is assumed to be constant and $\mathscr{F}_0$ is the information set available at time $t = 0$. Under the modelling framework assumed in this paper, the insurance premium can then be expressed as:

$$p_{0,t} = E\left[\int_0^t e^{-rs}\beta e^{\gamma s}\Lambda_s ds|\Lambda_0\right]. \tag{2.5}$$

Since an adaptation investment project helps to reduce the loss frequency, and therefore the insurance premium by a proportion k, the discounted value of the project that is invested at time t, is:

$$E\left[\int_t^\infty e^{-rs}k\beta e^{\gamma s}\Lambda_s ds|\mathscr{F}_t\right]. \tag{2.6}$$

Although (2.6) suggests that the value of the project depends on the Poisson intensity as well as the loss exposure, which in turn depends on time due to exposure growth, these two state variables can be summarized by the insurance premium flow $\pi_t = \beta e^{\gamma t}\Lambda_t$. Using Ito's Lemma, it is straightforward to show that π_t follows a GBM of the form:

$$d\pi_t/\pi_t = (\mu+\gamma)dt + \sigma dW_t. \tag{2.7}$$

Since $E[\pi_s|\pi_t] = \pi_t e^{(\mu+\gamma)(s-t)}$ for all $s > t$, the current value of the project invested at time t, conditional on $\pi_t = \pi$, is given by

$$V(\pi) = \frac{k\pi}{r-\mu-\gamma}. \tag{2.8}$$

The value of the project $V(\pi_t)$ obtained in (2.8) depends on the value of π_t observed at time t, and as π_t grows at rate $\mu+\gamma$, $V(\pi_t)$ also grows at rate $\mu+\gamma$. The rate $\mu+\gamma$ may be lower than the equilibrium rate of return that investors require to hold a risky asset like the investment project (which is r since the price of climate risk is zero). This is actually rather normal for real assets. In our case, the growth rate of $V(\pi_t)$ is partly driven by climate change which should not be affected by investors' activities in financial markets. For some investment projects such as those in high technologies industry, the growth rates of the project values may be even negative due to the growing competition after the new product is introduced (McDonald & Siegel, 1986). The value of the project is similar to the value of a stock that pays dividend at rate $r-(\mu+\gamma)$, and to value the investment option, we will use the approach discussed by Merton (1973) for pricing an American option on a dividend paying stock. Our project value plays the role of the stock price and the growth rate $\mu+\gamma$ plays the role of the required rate of return in the risk neutral martingale measure of the dividend paying stock, i.e., $\mu+\gamma = r-\delta$, where δ is the dividend payment rate of the stock. Merton (1973) shows that when δ is zero, it is never optimal to exercise the option, and for our investment problem to have a solution, we need $\mu+\gamma$ to be lower than r. Otherwise, the value of the investment opportunity would be infinite and it is never optimal to invest.

To determine the optimal investment decision, let $F(\pi)$ be the value of the option to invest when $\pi_t = \pi$. Then, given the observed value of π_t at any time t, an investment decision is optimal if it provides the maximum value for the project. Immediate investment yields the net present value of the project $V(\pi_t) - (I + C/r)$, while deferring the investment to the next instant $t+\Delta t$ gives a value $e^{-r\Delta t}E[F(\pi_{t+\Delta t})]$.[2] The option value $F(\pi_t)$ is the maximum of the net present value obtained from immediate investment and the value obtained by deferring the project, i.e.,

$$F(\pi_t) = \max\left\{V(\pi_t) - (I + C/r), e^{-r\Delta t}E[F(\pi_{t+\Delta t})]\right\}. \tag{2.9}$$

[2] Note that the risk free rate is used to discount the option value since the risk involved with the investment project is non-systematic. For a detailed discussion on the selection of the discount rate, see e.g., McDonald and Siegel (1986).

The option value defined in (2.9) is similar to the value of an option written on a financial stock. Hereby, $V(\pi_t)$ plays the role of the stock price and $I + C/r$ plays the role of the strike price. Using Ito's Lemma, (2.9) can be expressed as

$$\max\left\{V(\pi_t) - (I+C/r) - F(\pi_t),\; \frac{1}{2}\sigma^2\pi_t^2F''(\pi_t) + (\mu+\gamma)\pi_t F'(\pi_t) - rF(\pi_t)\right\} = 0. \tag{2.10}$$

The option value then satisfies the stochastic differential equation:

$$\frac{1}{2}\sigma^2\pi_t^2F''(\pi_t) + (\mu+\gamma)\pi_t F'(\pi_t) - rF(\pi_t) = 0, \tag{2.11}$$

with boundary conditions:

$$F(\pi_t) = V(\pi_t) - (I + C/r) \tag{2.12}$$

$$F'(\pi_t) = V'(\pi_t). \tag{2.13}$$

The value of the option is then given by

$$F(\pi_t) = B\pi_t^\alpha, \tag{2.14}$$

where:

$$\alpha = \frac{1}{2} - \frac{\mu+\gamma}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu+\gamma}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{2.15}$$

$$B = \left(\frac{k}{r-\mu-\gamma}\right)^\alpha \frac{(\alpha-1)^{\alpha-1}}{\alpha^\alpha}(I+C/r)^{1-\alpha}. \tag{2.16}$$

The option is exercised when π_t is greater or equal to the investment threshold given by:

$$\pi^* = \frac{\alpha}{\alpha-1}\frac{r-\mu-\gamma}{k}(I + C/r). \tag{2.17}$$

2.3. Choosing among alternative investment projects

Let us now assume that the decision maker can invest in only one of n projects. We assume that each project i has investment cost I_i, a maintenance cost flow C_i and reduces the loss frequency by a proportion k_i, $i = 1, \ldots, n$. This means that at any time s, project i helps to reduce the insurance premium flow π_s by the proportion k_i. At time t, if the decision maker decides to invest in project i, he/she will obtain the project i with value:

$$V_i(\pi) = \frac{k_i\pi}{r-\mu-\gamma}. \tag{2.18}$$

The investment problem is that at any time t, the decision maker observes the value π_t and determines whether to invest in one of the n projects or to defer the investment to a later time. If the decision maker decides to invest in project i, he/she gets the net present value of project i, $V_i(\pi_t) - (I_i + C_i/r)$, and the decision process stops. If the decision maker decides to wait, then at a later point in time $t+\Delta t$, he/she can consider the decision of whether to invest in one of the n projects or to defer the investment again. As a result, the value of the option to invest is the maximum of the values of the individual projects and the value of deferring the investment. In other words, if $F(\pi_t)$ is the value of the option to invest in one of the n projects, then $F(\pi_t)$ satisfies:

$$F(\pi_t) = \max\{V_1(\pi_t) - (I_1 + C_1/r), \ldots, V_n(\pi_t) - (I_n + C_n/r), e^{-r\Delta t}F(\pi_{t+\Delta t})\}. \tag{2.19}$$

The solution of Eq. (2.19) depends on the existence of a dominant project among the n projects that are considered. Fig. 1 illustrates the situation, where a dominant project exists (*Panel a*)

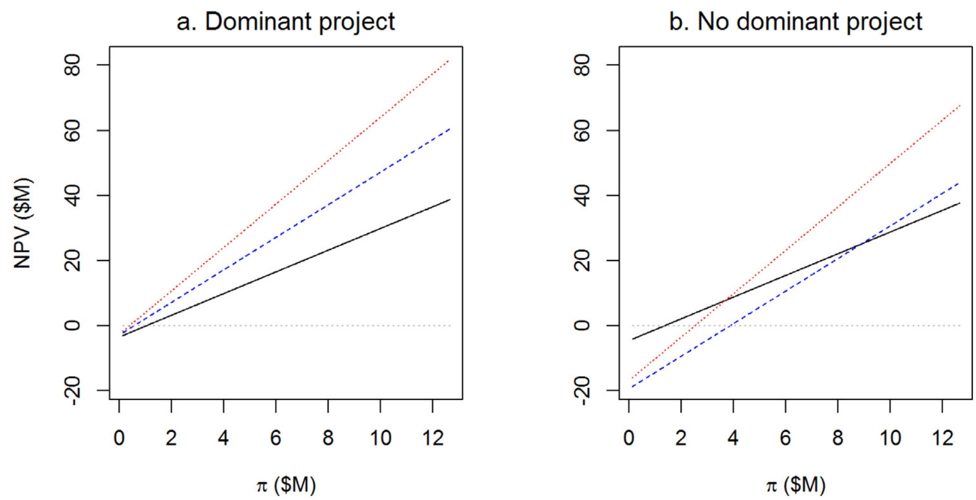

Fig. 1. NPV profiles of different projects. A project with a NPV always higher than the positive NPV of all other projects (for any choice of π) is a dominant project. For example, the project in *Panel a*, where the NPV is represented by the dotted line is a dominant project. *Panel b* illustrates the situation where no dominant project exists. Different projects yield the highest NPV for different ranges of insurance premium flow π.

as well as the situation where different projects yield the highest NPV for different values of π (*Panel b*). When a dominant project m exists, i.e., there is a project whose NPV is higher than the positive NPV of all other projects for any choice of π, Eq. (2.19) reduces to

$$F(\pi_t) = \max\left\{V_m(\pi_t) - (I_m + C_m/r), e^{-r\Delta t}F(\pi_{t+\Delta t})\right\}, \tag{2.20}$$

and the option to invest in one of the n projects reduces to the option to invest in project m. The solution to this investment problem is provided in Section 2.2.

When no dominant project exists, as illustrated in *Panel b* of Fig. 1, the optimal investment rule may include investing in a non-dominated project when the state variable π_t is in a low range and investing in another non-dominated project when the state variable is in a high range. Suppose that there are two non-dominated projects, where Project 1 is dominant for a lower range and Project 2 dominates for a higher range of values for the state variable π_t. For a value of π_t close to zero, the optimal decision is not to invest in any of the projects. For such a scenario, since the optimal decision is to wait, the value of the option to invest is given by:

$$F(\pi_t) = B\pi_t^{\alpha}, \tag{2.21}$$

where α is given in (2.15) and B is a parameter to be determined. Since the decision maker can choose to invest in either of the projects at any time, the option to invest in one of the two projects is the higher of the values of the option to invest in each individual projects, $F_1(\pi_t)$ and $F_2(\pi_t)$. This is satisfied if $B = \max_i B_i, \quad i \in \{1, 2\}$, where $B_i = \left(\frac{k_i}{r-\mu-\gamma}\right)^{\alpha} \frac{(\alpha-1)^{\alpha-1}}{\alpha^{\alpha}}(I_i + C_i/r)^{1-\alpha}$.

If $B_2 > B_1$ or equivalently,

$$\frac{k_2^{\alpha}}{(I_2 + C_2/r)^{\alpha-1}} > \frac{k_1^{\alpha}}{(I_1 + C_1/r)^{\alpha-1}}, \tag{2.22}$$

then the value of the option to invest in alternative projects, $F(\pi_t)$, is equal to the value of the option to invest in Project 2, $F_2(\pi_t)$. The optimal decision is to wait for investment in Project 2 while the state variable π_t is lower than π_2^* (the investment threshold for Project 2) and to invest in Project 2 when π_t is higher than π_2^* (*Panel a* in Fig. 2). Thus, in this case, the option to invest in one of the two projects is the same as the option to invest in Project 2.

On the other hand, if $B_2 < B_1$, then a low value for π_t yields $F_2(\pi_t) < F_1(\pi_t)$ and the value of the option is equal to the value of the option to invest in Project 1 (*Panel b* in Fig. 2). It is optimal to wait while π_t is lower than the optimal investment threshold of Project 1, π_1^*. When π_t is higher or equal to π_1^*, and lower than a level $\pi_t^{\dagger}$, it is optimal to invest in Project 1 immediately. When π_t is above $\pi_t^{\dagger}$ and lower than the optimal investment threshold for Project 2, π_2^*, it is optimal to wait for investing in Project 2. For π_t above π_2^*, immediate investment in Project 2 is optimal. The only difference between this problem and the problem of investing in an individual project is that we need to determine $\pi_t^{\dagger}$.

Note that at $\pi_t^{\dagger}$, the decision maker is indifferent between immediate investment in Project 1 and waiting for investment in Project 2, i.e., $V_1(\pi_t^{\dagger}) - (I_1 + C_1/r) = F_2(\pi_t^{\dagger})$, and therefore $\pi_t^{\dagger}$ can be found by solving equation:

$$\frac{k_1\pi_t^{\dagger}}{r-\mu-\gamma} - (I_1 + C_1/r) = B_2\pi_t^{\dagger\alpha}. \tag{2.23}$$

The value of the option to invest in alternative projects is equal to the value of the option to invest in Project 1 when π_t is lower than π_1^*; equal to the value of Project 1 when π_t is between π_1^* and $\pi_t^{\dagger}$; equal to the value of the option to invest in Project 2 when π_t is between $\pi_t^{\dagger}$ and π_2^*; and equal to the value of Project 2 when π_t is above π_2^*.

2.4. Sequential investment

So far in the assumed setting with alternative investment projects, although we consider many projects at the same time, we have restricted the decision maker to invest in only one project over the whole time horizon. However, this would only be the case if the decision maker is subject to a restricted budget and therefore can afford to invest in one project only. When the budget constraint is relaxed, it may be optimal to invest in additional projects at a later point in time after the initial investment, if the state variable π_t increases to a sufficiently high level. In comparison to considering alternative investment projects only, sequential investment results in higher benefits (for the application of the real options approach versus the NPV rule) due to more investment opportunities. For simplicity and to illustrate the approach, in the following we consider two projects for sequential investment. However, an extension of the approach to sequential investment with a higher number of projects is straightforward.

Suppose that Project 1 is invested first, and when π_t is sufficiently high, Project 2 is invested. Then, the investment problem

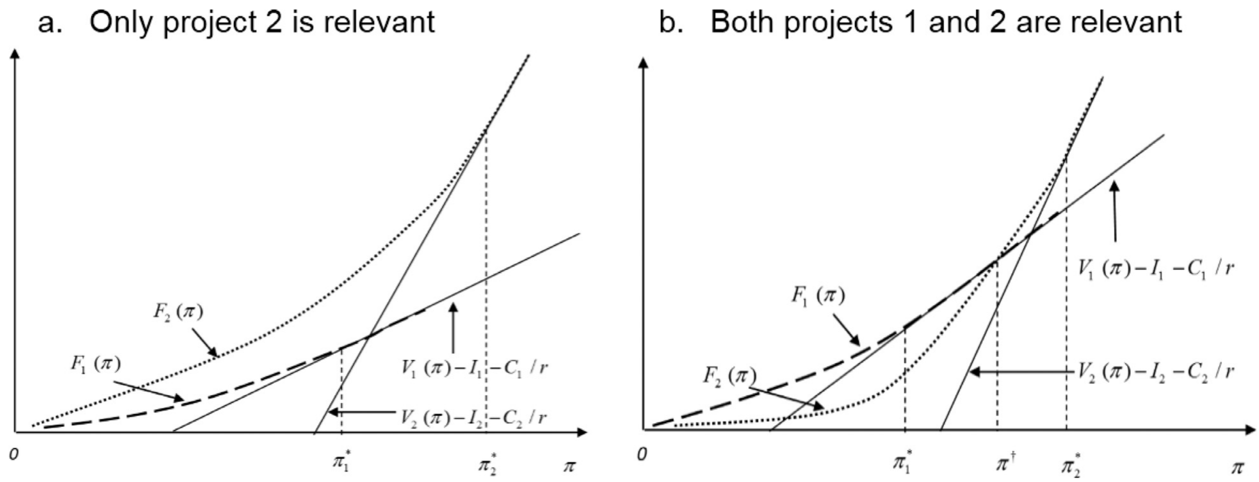

Fig. 2. Relevance of investment projects when no project is dominant. *Panel a* demonstrates that Project 1 becomes irrelevant when it is dominated in a higher range of π and also has a lower option value (compared to that of Project 2) in the range of π where waiting to invest in Project 1 is optimal. *Panel b* demonstrates that Project 1 is still relevant to the investment problem when it is dominated in a higher range of π, but has a higher option value (compared to that of project 2) in the range of π where waiting to invest in Project 1 is optimal.

can be denoted by

$$F_1(\pi_t)+F_{12}(\pi_t)=\max_{\tau_1,\tau_2}E\left[\int_{\tau_1}^{\infty}[k_1\pi_s-C_1]e^{-rs}ds-e^{-r\tau_1}I_1+\int_{\tau_2}^{\infty}[k_2(1-k_1)\pi_s-C_2]e^{-rs}ds-e^{-r\tau_2}I_2|\pi_t\right], \tag{2.24}$$

where $F_{12}(\pi_t)$ is the value of the option to invest in Project 2, after Project 1 has already been invested. Similarly, if Project 2 is invested first, then $F_2(\pi_t)+F_{21}(\pi_t)$ is obtained. In considering which project to be invested first, the decision maker needs to select a sequence that maximizes the option to invest, $F_s(\pi_t)$, i.e.,

$$F_s(\pi_t)=\max\{F_1(\pi_t)+F_{12}(\pi_t),F_2(\pi_t)+F_{21}(\pi_t)\}. \tag{2.25}$$

For a value of π_t close to zero, the values of the options are as in (2.14), with $k_{12}=k_2(1-k_1)$ and $k_{21}=k_1(1-k_2)$. From the expression of option values, it is optimal to invest in Project 1 first, if:

$$\frac{k_1^\alpha}{(I_1+C_1/r)^{\alpha-1}}+\frac{((1-k_1)k_2)^\alpha}{(I_2+C_2/r)^{\alpha-1}}>\frac{k_2^\alpha}{(I_2+C_2/r)^{\alpha-1}}+\frac{((1-k_2)k_1)^\alpha}{(I_1+C_1/r)^{\alpha-1}}. \tag{2.26}$$

After the optimal sequence of investments has been identified, investments are conducted using the optimal investment thresholds:

$$\pi_i^*=\frac{\alpha}{\alpha-1}\frac{r-\mu-\gamma}{k_i}(I_i+C_i/r), \tag{2.27}$$

where $i\in\{1,2,12,21\}$ and $I_{12}=I_2,I_{21}=I_1,C_{12}=C_2,C_{21}=C_1$, $k_{12}=(1-k_1)k_2$, $k_{21}=(1-k_2)k_1$. Clearly, the illustrated framework for sequential investment can be easily extended to a situation where the optimal sequence of investments involves more than two projects.

It is worth to note that in our setup, the major impact of maintenance costs is the increase of the effective investment costs for the projects. Maintenance costs may have other impacts on investment decisions when the option to exit the project or the option to temporarily suspend the project is considered. In these cases, exiting or suspending the project helps to avoid the maintenance cost and reduces the commitment to the project cost. This effectively reduces the investment cost of the project and leads to earlier investment. However, the impact of an exit option will typically be rather small since the prospect of exiting the project is likely to relate to the far future. In a similar study, Décamps et al. (2006) also ignore the option to exit when they examine alternative investment projects. Further, temporarily suspending risk reduction projects may not be feasible, since the projects typically relate to public infrastructure and the government may be required to maintain the project continuously (Ku-ring-gai Council, 2010b).

2.5. Loss due to using suboptimal investment rules

Consider an individual investment project with real options value $F(\pi_t)$ and an optimal investment threshold π^*. We will examine the loss incurred when the project is invested based on suboptimal investment rules such as a NPV rule or an investment rule given by a deterministic model.

For the NPV rule, it is well known that the optimal investment threshold π^* is higher than the value π_N at which the NPV of the project, $[V(\pi_N)-(I+C/r)]$, is zero, and that at the optimal investment threshold, the NPV of the project is positive, see e.g., Dixit and Pindyck (1994).[3] When π_t is higher than π_N, the project is invested under the NPV rule and the loss due to using the NPV rule is the difference between the option value and the NPV, i.e., $F(\pi_t)-[V(\pi_t)-(I+C/r)]$. In contrast, when π_t is lower than π_N, the project has a negative NPV and is, therefore, not invested under the NPV rule. However, in this case, the loss due to the use of the NPV rule is not zero since the project will be suboptimally invested whenever π_t reaches π_N at a future time.

The expected loss at a value $\pi_t<\pi_N$ due to the use of the NPV rule is the value of the option[4] at π_N discounted by the time τ that π_t takes to reach π_N from its current level π_t:

$$F(\pi_N)E[e^{-r\tau}]. \tag{2.28}$$

As demonstrated by Dixit and Pindyck (1994, p.315), $E[e^{-r\tau}]=(\pi_t/\pi_N)^\alpha$, where α is given in (2.15). Since $F(\pi_N)=B\pi_N^\alpha$, the loss is then $B\pi_t^\alpha$, which is the value of the option to invest. In summary, the loss due to the use of the NPV rule is therefore equal to the value of the option for negative NPV projects and equal to the

[3] With π^* given in (2.17), the NPV of the project when $\pi_t=\pi^*$ is $(I+C/r)/(\alpha-1)$ which is positive since $\alpha>1$. For the proof that $\alpha>1$, see Dixit and Pindyck (1994, p.143).

[4] When π_t reaches π_N, the loss is the difference between the option value and the NPV of the project. However, by definition of π_N, the NPV of the project is zero and therefore, the loss is the same as the option value at π_N.

difference between the option value and the NPV for positive NPV projects.

In a deterministic model, it is assumed that $\sigma = 0$ and $\pi_t = \pi_0 e^{(\mu+\gamma)t}$. In this model, the project is invested when the NPV of the project reaches the maximum level. The NPV of the project invested at time t is defined as:

$$NPV_t = \int_t^{\infty} e^{-rs} k\pi_s ds - e^{-rt}(I + C/r)$$
$$= \frac{k\pi_0}{r-\mu-\gamma} e^{-(r-\mu-\gamma)t} - e^{-rt}(I + C/r), \tag{2.29}$$

which is maximized when the project is invested at time $t^* = \frac{1}{\mu+\gamma} \ln \frac{rI+C}{k\pi_0}$, or when π_t reaches a threshold $\pi_D = \frac{rI+C}{k}$. Since catastrophic risk is stochastic in reality, the investment value obtained when using the deterministic model is the NPV of the project that is obtained when π_t reaches π_D discounted by the wating time τ.[5] When $\pi_t > \pi_D$, no waiting is required and the loss is equal to the difference between the option value and the NPV, $F(\pi_t) - [V(\pi_t) - (I + C/r)]$. When $\pi_t < \pi_D$, the expected loss is $F(\pi_t) - [V(\pi_D) - I - C/r]E(e^{-r\tau})$. Since $E(e^{-r\tau}) = (\pi_t/\pi_D)^{\alpha}$, the expected loss is:

$$B\pi_t^{\alpha} - \frac{\mu+\gamma}{r-\mu-\gamma}(I + C/r)(\pi_t/\pi_D)^{\alpha}, \tag{2.30}$$

which can be expressed in the following form after using $\pi_D = \frac{rI+C}{k}$ and the expression of B in (2.16):

$$\left[\alpha^{-\alpha}\left(\frac{\alpha-1}{r-\mu-\gamma}\right)^{\alpha-1} - \frac{\mu+\gamma}{r^{\alpha}}\right]\frac{(I + C/r)^{1-\alpha}(k\pi_t)^{\alpha}}{r-\mu-\gamma}. \tag{2.31}$$

3. Case study analysis

In this section, we apply the proposed framework to a case study of bushfire risk management in a local government area (Ku-ring-gai) in Southeastern Australia. This is an urban area that has residential properties surrounded by large national parks. It has a substantial interface between bushland and urban land (89 kilometers) and is the third most vulnerable local government area with regards to bushfire risk in Sydney (Chen, 2005).[6]

Main adaptation options to reduce bushfire risk in the region include building new fire-trails, constructing new rural fire-stations and land rezoning (Ku-ring-gai Council, 2010a). Fire trails allow more effective controlled hazard reduction burning to be carried out, break the transition of wild fire and also provide longer time for rural fire fighters to respond before a fire develops beyond suppression. Building additional rural fire stations equips volunteer rural fire fighters with sufficient fire suppression equipment and helps to respond to bushfires more timely and effectively. In the following, we will consider a situation, where two alternative adaptation investment projects are being examined: an investment project that provides additional fire trails and an investment project that constructs an additional rural fire station. We will use this case study to illustrate the application of the modelling framework and to provide economic insights on the value of investment flexibility.

The benefits generated by the adaptation projects may not be limited to the reduction in bushfire risk. If properly designed, the projects can have co-benefits that are independent of the evolution of bushfire risks. For example, fire trails can be used for cycling, hiking and other recreational activities while rural fire stations can be used for the mitigation of other risks such as storm and flood risk or for search and rescue activities.[7] Usually, co-benefits are not transacted values observed in markets and they are often difficult to quantify and monetize. We will outline how the modelling framework can be extended to incorporate such co-benefits of a project in Section 3.4.

3.1. Parameter calibration

Bushfire risk in Australia is highly impacted by climate variables, especially temperature and wind speed, and is predicted to increase significantly in future years due to the impact of climatic change (Lucas, 2010). Reliable forecasts about how the climate will change, are, however, difficult to obtain, since the change depends on, e.g., carbon emissions with a significant time lag, while predictions based on historical climate observations may be unsatisfactory (Matsumoto & Andriosopoulos, 2016; Wei, Mi, & Huang, 2015). Studies that forecast climate change usually utilize climate models that simulate the interactions among important drivers of climate, including atmosphere, oceans, land surface and ice (Solomon, 2007). There are, however, many climate models and their reliability is usually difficult to measure. In this paper, we rely on climate change impact studies to calibrate the Poisson intensity process Λ_t.

The ideal condition for calibration is when there exist many climate change impact studies that provide not only point forecasts, but also distributional forecasts. This condition is satisfied for the case of sea level rise, and Mills et al. (2014) have calibrated a real options model so that the modelled sea level rise distribution at a future time is the same as the weighted distribution given by climate change studies, with the weight determined by the credibility of the authors or research institutions.

For bushfires, there are, however, not many studies that provide distributional forecasts for future bushfire risk. An exception is the study by Pitman, Narisma, and McAneney (2007) who use one model, the regional atmospheric modelling system developed by the Colorado State University, and two emission scenarios (A2 and B2) to provide 95% confidence intervals for a fire weather index for Australia. Knowing the distribution of the fire weather index for the next (approximately) 100 years and the current value of the index, we can calculate the distribution for the growth rate of the fire weather index and also the parameters μ and σ for the Poisson intensity process. Based on the results provided in Table 1 in Pitman et al. (2007), we obtain an annual volatility of 7%, which is quite low compared with the volatility we obtain using a point forecast study. This low volatility is probably due to the low number of climate models used by Pitman et al. (2007) and also the low number of simulations (four for each emission scenarios) being used to obtain the distributional forecast. We therefore use the results of a point forecast study by Hasson et al. (2009) to calibrate the Poisson intensity process, but we acknowledge that a more precise calibration of the model could be based on results from distributional forecasts.

Hasson et al. (2009) use 10 general circulation models and two GHG emission scenarios, a low (B1) and a high (A2) emission scenario, to forecast the frequency of extreme fire weather events in Southeastern Australia. The forecast exercise yields 20 point forecasts, with some models predicting the expected frequency of bushfire to grow as much as 6% per year while some models predicting decreases in bushfire risk. We assume that each model uses different radiative forcing parameter values that are equally proba-

[5] Note that the value $[V(\pi_D) - I - C/r]e^{-rt^*}$ that is obtained by substituting t^* into (2.29) and is usually given by deterministic models would not correctly state the investment value obtained by exercising the investment option at threshold π_D. It understates the actual value since $e^{-rt^*} < E(e^{-r\tau})$. The last inequality is a result of the convexity of the exponential function and the fact that t^* is close to (but larger than) the expected value of τ.

[6] Vulnerability to bushfire risk is measured by the number of properties within 130 meters of bushland.

[7] We would like to thank a referee for pointing this out.

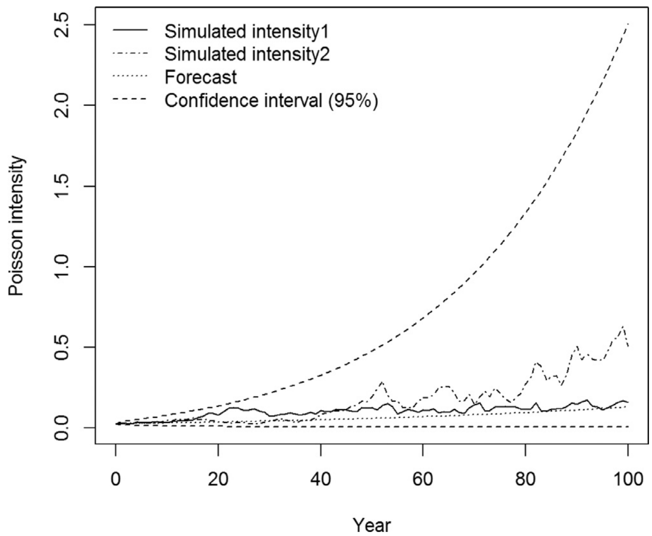

Fig. 3. Estimated process of bushfire intensity Λ_t. We use 20 alternative forecasts generated from 10 general circulation models using a low (B1) and a high (A2) GHG emission scenario suggested in Hasson et al. (2009) to obtain estimates for expected growth rate $\mu = 0.0159$ and volatility $\sigma = 0.1503$ for the process Λ_t. The figure also illustrates two simulated paths as examples for the dynamics of bushfire intensity over the considered time horizon.

ble and that the two emission scenarios are also equally likely. The 20 forecasts then form an empirical distribution for Λ_T and given the current value Λ_0, the annual growth rate for the frequency of extreme bushfire events can be calculated from these predictions. Based on the provided forecasts, the estimates for the parameters of the Poisson intensity process are $\mu = 0.0159$ and $\sigma = 0.1503$. The estimated process for Λ_t is illustrated in Fig. 3.

To monitor the value of the Poisson intensity Λ_t, we use an estimated Poisson regression model based on bushfire events observed in NSW and weather data collected at Sydney airport, NSW. The dataset is available on a daily basis and spans the period 1970–2008. The regression results are as follows:

$$P(Y_t = y) = \frac{(\Lambda_t)^y e^{-\Lambda_t}}{y!},$$
$$\log \Lambda_t = \underset{(2.43)}{-14.3520} + \underset{(0.05)}{0.2637} \times T\max_t + \underset{(0.02)}{0.0703} \times \text{Wind}_t - \underset{(0.02)}{0.0428} \times \text{Humid}_t \tag{3.1}$$

where Y_t is the number of bushfire events occurring on day t, Λ_t is the intensity parameter that controls the probability of a catastrophic event on day t, $T_{\max t}$ is the maximum temperature on day t, Wind_t is the wind speed at 3 p.m. on day t and Humid_t is the average humidity on day t. The numbers in brackets are the standard errors of the estimated coefficients. Using t-tests, we find that the maximum temperature and wind are significant at the 1% level, while humidity is significant at the 5% level. The model has a pseudo-R squared of 23%, which measures the improvement in the log-likelihood of the model that includes the considered explanatory variables in comparison to a model without covariates.

We obtain the Poisson intensity for Ku-ring-gai by downscaling the risk estimated for NSW. In the Ku-ring-gai region, there was only one house-damaging bushfire event over the sample period (38 years), and the average probability of an event occurring in one year is $1/38 = 0.0263$. We therefore scale down the NSW Poisson intensity by a factor of 16 so that the scaled-down risk has an annual average of 0.0263. While the average value of the Poisson intensity is 0.0263, the realized values vary over time with the last observed value of 0.035. We will use the last observed value of the Poisson intensity as the estimate of its current value.

To determine the discount rate, we use the estimation results provided by Truong and Trück (2016a) who estimate the stochastic interest rate model proposed by Cox, Ingersoll, and Ross (1985), and use the expected discount factor based on a stochastic interest rate model to find the certainty equivalent discount rate. The estimated certainty equivalent discount rate is found to converge quickly to the long run level (4.5%), and for simplicity, we assume that the discount rate is constant at 4.5%.

The distribution of loss severity is estimated based on the assumption that if a house catches fire, it will be destroyed, what has also been suggested based on empirical studies, see, e.g., Crompton, McAneney, Chen, Pielke, and Haynes (2010). Then, the expected loss severity is a product of the expected number of damaged houses and the cost of reconstructing a house. The reconstruction cost is obtained as the difference between the property price and land price for the considered region.[8] The estimate for reconstruction cost per house is $405,000.

We obtain the expected number of damaged houses in a bushfire event from the information provided by a bushfire expert in the area. As suggested by the expert, for a severe bushfire, the average number of houses being damaged is 30. The expected loss without growth of loss exposure is then $12.15 million.

To estimate the investment costs, loss mitigation effectiveness and project life, we use the expert elicitation method that has been used in many previous climate adaptation studies, see e.g., Baker and Solak (2011); Mathew et al. (2012); Truong and Trück (2016a), to overcome the problem of data scarcity. The expert specifies that additional fire trails are expected to reduce the frequency of house damaging bushfire events by 20%, while investment in an additional fire station is expected to reduce the frequency of house damaging bushfire events by 18%. We assume that the risk reduction for the two adaptation investments is independent due to the different nature of the investments. While fire trails will lead to a reduced risk as a result of an increased and more effective hazard reduction burning program, a new fire station will allow a better protection of properties and infrastructure near a park.

The investment cost of an infinite lifetime project can be calculated from the investment cost I_M for a finite lifetime project estimated by the expert Table 1 by firstly converting I_M into an annuity flow, A:

$$A = I_M \frac{1 - (1+r)^{-1}}{1 - (1+r)^{-(M+1)}}.$$

Based on the annuity A, the investment cost of a project that lasts infinitely can be easily calculated:

$$I = A(1+r)/r. \tag{3.2}$$

At the discount rate of 4.5%, the present value of building a fire station every 40 years at the cost of $0.75 million each is $0.90 million while the present value of building bushfire trails every 50 years at the cost of $1.5 million each time, is $1.68 million.

3.2. Baseline case

In a first step we discuss the results for the baseline values of parameters obtained in the previous section. A summary of these results is provided in Table 2. Given the estimated parameters, the NPV of immediate adaptation investment in the fire station is $1,558,544, while the NPV for investment in the fire trails is $1,668,692. Should the NPV rule be used, the investment into the fire trails that yields a higher NPV would be preferred. After investment into the fire trails, bushfire risk is reduced by 20% and further investment in the fire station would give a NPV of $866,334.

[8] We use the average net-of-realtor-commission property sales price in the area provided by Hatzvi and Otto (2008) where the realtor commission is 2.5% and the land value provided by the NSW Valuer General (DOL, 2009).

Table 1
Information on estimated and assumed parameter values, including expected loss $E(X_0)$, the estimated growth rate of the cost of reconstruction γ, the current intensity of bushfires Λ_0, the expected intensity growth rate μ and the volatility of intensity growth rate σ. For each type of projects (fire trails vs. rural fire station), the table also provides information about the assumed risk mitigation impact of the project k, the life span of each project M, the investment cost per project I_M, the maintenance costs C of the project and the applied discount rate r.

Parameters	Value
Current Poisson intensity (Λ_0)	0.035
Expected rate of Poisson intensity growth (μ)	1.59%
Volatility of Poisson intensity (σ)	15.03%
Expected loss severity ($E(X_0)$)	\$12.15 million
Growth rate of reconstruction cost (γ)	1%
Risk mitigation by the fire station (k_1)	18%
Risk mitigation by the fire trails (k_2)	20%
Lifetime of the fire station (M_1)	40 years
Lifetime of the fire trails (M_2)	50 years
Investment cost per project for rural fire station (I_M^1)	\$0.75 million
Investment cost per project for fire trails (I_M^2)	\$1.5 million
Maintenance cost of rural fire station (C_1)	\$70,000
Maintenance cost of fire trails (C_2)	\$50,000
Discount rate (r)	4.5%

Since the NPV of the fire station is positive, it would be invested according to the NPV rule. Thus, using the NPV rule to guide investment, both the fire trails and the fire station would be invested immediately with an overall NPV for investing in both projects at time $t = 0$ of \$2,424,877.

Alternative to the NPV rule, a deterministic model as described in Section 2.5 can be used. The deterministic model makes use of the flexibility that investment can be deferred to the years when the annual insurance premium becomes sufficiently large to avoid the high capital expenses in the initial years. The deterministic model would increase the value obtained from the fire station project from \$1,558,544 (given by the NPV rule) to \$1,959,025 while increasing the investment value of the fire trails to \$2,154,372 (from \$1,668,692 when the NPV rule is used). Therefore, if only one project can be financed, the fire trails project would be selected, and invested when the annual insurance premium rises to \$627,492 (from the current premium of \$425,250). In contrast, when two projects can be financed, investing into the fire station first and the fire trails later will give a higher value. The fire station will be invested when the annual insurance premium reaches \$613,312 and after the investment of the fire station, the fire trails will be invested when the premium reaches \$765,235. As illustrated in Table 2, the deterministic model increases the value obtained from the investment sequence to \$3,574,376 (from \$2,424,877 when the NPV rule is used).

To apply the real options model for investing in a single project when multiple projects are available, we depict the NPV profiles of the two projects in Fig. 4. The fire station dominates when the

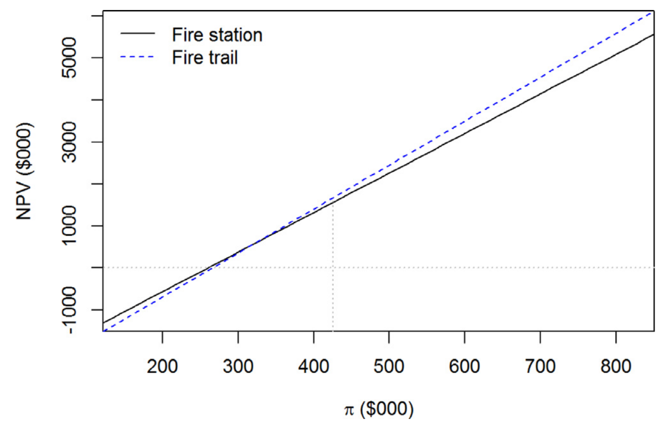

Fig. 4. NPV profiles of investment into a fire station (solid) and a fire trail (dashed) adaptation project for different values of π_t. The fire station dominates when π_t is lower than \$315,900, otherwise the fire trail investment is preferred.

annual insurance premium (π_t) is lower than \$315,900, otherwise the fire trail investment dominates. In addition, since condition (2.22) is satisfied, the option to invest in the fire trails is higher than the option to invest in the fire station for all π_t (despite the dominance of the NPV profile of the fire station for $\pi_t <$ \$315,900). If the decision maker has to choose between investment into fire trails or the fire station, the fire trails project would be selected. The value of the option to invest in alternative projects is then equal to the value of the option to invest in the fire trails (\$2,240,858). It is optimal to invest in the fire trails when the annual insurance premium is equal to or higher than $\pi_2^* =$\$856,022.

In contrast, if the decision maker can invest in the fire trails as well as the fire station in a sequential order, he/she should invest in the fire station first when the annual insurance premium reaches \$836,677 and invest in the fire trails when the premium reaches \$1,043,929. At the current value of the insurance premium (\$425,250), the value of the option to invest in the fire station first and the fire trails later is \$3,717,868, while the value of the option to invest in the sequence of fire trails first and the fire station later is \$3,714,935. As expected, the value of the sequential investment option is less than the sum of the individual investment options (\$4,278,527), due to the fact that after a project has been invested, there remains less risk to be mitigated.

Using the NPV rule would result in significant losses in option values, especially when multiple projects can be afforded. For the case of alternative project investment where the fire trails are selected over the fire station, a loss of \$572,166 or 25.53% of the option value would result. For the case where multiple projects can be invested, using the NPV rule would incur a loss of \$1,292,991 or 34.78% of the sequential investment option value. The NPV rule

Table 2
Results for baseline case for different investment settings. Investment can be conducted individually into one of the projects, i.e., a new fire station or fire trails according to the NPV rule at $t = 0$ or according to the decision rules provided by the deterministic model or the real options model. Alternatively, sequential investment (either $F_1 + F_{12}$ or $F_2 + F_{21}$) can be conducted, taking into account the optimal sequential timing for each investment.

	Investment model		
Investment value	NPV rule (\$)	Deterministic model (\$)	Real options (\$)
Individual investment			
F_1 (Fire station)	1,558,544	1,959,025	2,037,669
F_2 (Fire trails)	1,668,692	2,154,372	2,240,858
Combined/sequential investment			
$F_1 + F_{12}$	2,424,877	3,574,376	3,717,868
$F_2 + F_{21}$	2,424,877	3,571,556	3,714,935

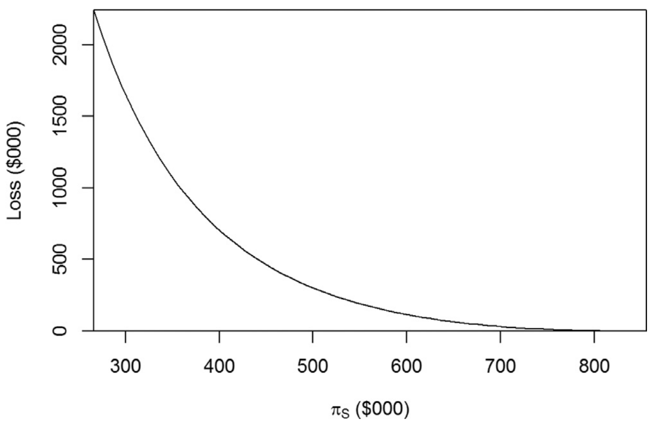

Fig. 5. Loss due to exercising the option to invest in alternative projects at threshold $\pi_S < \pi_2^*$. The investment thresholds for the NPV rule, the deterministic model and the real options model are $266,057, $627,492 and $856,022, respectively.

results in large losses in the option values because the NPV rule advises to exercise the option when the option is at the money, i.e., when the difference between the option value and the NPV of the project is largest. As illustrated in Fig. 5 for the case of alternative projects, as the suboptimal investment threshold π_S increases, the loss decreases at increasing rates to zero for the case when the current insurance premium is lower than the suboptimal investment threshold π_S.

Using the deterministic model would help to reduce the losses significantly. The loss in alternative investment option value would be reduced from 25.53% as in the case of the NPV rule to 3.86% ($86,486) while the loss in the sequential investment option value would be reduced from 34.78% to 3.86% ($143,492). Although a deterministic model is much better than the NPV rule, the investment decision can still be improved by using a real options model.

3.3. Sensitivity analysis

In the following, we examine the sensitivity of the results to changes in key parameters of the model. In a first step, we increase the value of each parameter by 10% to examine the impact of such a change on the results. We then vary each parameter over a wide range of possible values to detect important trends.

When uncertainty, the risk growth rate (representing the magnitude of climate change), the discount rate, investment cost or expected loss severity increase by 10%, neither the selected project for alternative investment, nor the order of the invested projects in a sequential investment decision are changed. The resulting changes in the investment values given by different investment models and the changes in the loss due to the use of a suboptimal investment model (NPV rule, deterministic model) are presented in Table 3.

As can be seen, the discount rate is the parameter resulting in the largest changes to the investment values for all models and also affects the NPV of sequential investment the most. In addition, changes in the severity of climate change and the expected severity of catastrophic losses also result in large changes to the investment values provided by the three models. Changes in investment costs result in large changes to the NPV of the projects, but do not have a large impact on the investment values given by the deterministic model or the real options model. This is due to the fact that for these models, investment occurs in the future and the impact of changes in investment costs is discounted.

The losses that occur when the NPV rule is used are most sensitive to the expected loss severity and investment costs. This implies that as the scale of investment projects rises, it is increasingly important to use the deterministic model and a real options model rather than the NPV rule to guide investment decisions.

The losses caused by the suboptimality of the deterministic model are most sensitive to changes in the uncertainty, discount rate and expected loss severity. As the uncertainty parameter increases by 10%, the loss of the deterministic model in comparison to using a real options model increases by around 28%. This could be expected, since the deterministic model does not take into account the uncertainty of the frequency of loss events. Losses due to the deterministic model are also highly sensitive to the discount rate. As the discount rate increases by 10%, the losses decrease by approximately 15%. These results suggest that in particular for low discount rates, it is important to replace the deterministic model with a real options model. A lower interest rate reduces the annual costs of investment projects and results in earlier investment decisions in the deterministic model, making the investment decision by the deterministic model even more suboptimal (for more details, see Truong & Trück (2016a)). Losses due to using deterministic instead of a real options model are also sensitive to the expected loss severity: losses increase by around 15% when the expected loss severity increases by 10%. In contrast, an increase in the investment cost by 10% will reduce the loss by 2–3%. This means that when the scale of a project is increased by 10%, i.e., both the value of the project and investment cost are increased by 10%, the loss due to the use of a deterministic model will increase

Table 3
Sensitivity of the results for the investment values, the loss by using the NPV rule, and the loss by using the deterministic model (compared to the real options model) to a 10% increase in the value of considered key variables. The uncertainty parameter $\sigma = 15.03\%$, the discount rate (4.5%), investment costs ($0.75 million for fire station and $1.5 million for fire trails), and expected loss severity are increased by 10%. For the climate change variable, we assume that the expected growth rate of the Poisson intensity $\mu = 1.59\%$ is increased by 10% to $\mu = 1.75\%$.

	Δ Investment value			Δ Loss	
10% increase	NPV rule (%)	Det. model (%)	Real options (%)	NPV rule (%)	Det. model (%)
Alternative investment					
Uncertainty	0.00	1.18	2.20	8.63	27.72
Climate change	24.32	17.97	17.05	−4.14	−5.91
Discount rate	−42.63	−31.27	−30.63	4.36	−14.82
Investment cost	−10.05	−2.60	−2.60	19.13	−2.60
Expected severity	26.71	14.83	14.83	−19.82	14.83
Sequential investment					
Uncertainty	0.00	1.42	2.44	7.02	28.02
Climate change	28.78	18.41	17.48	−3.71	−5.56
Discount rate	−47.62	−31.29	−30.66	1.16	−14.85
Investment cost	−10.62	−2.06	−2.06	14.00	−2.06
Expected severity	31.62	14.83	14.83	−16.65	14.83

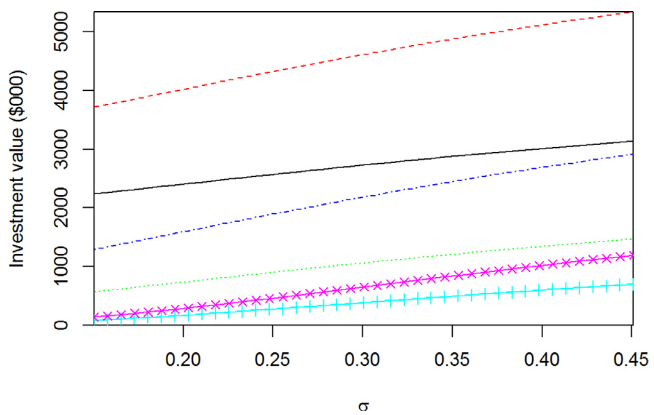

Fig. 6. Impact of different uncertainty parameter values ($0.15 < \sigma < 0.45$) on option values and the loss due to using sub-optimal investment models. We plot the investment option value for alternative projects ( ), the option value of sequential investment ( ), the loss due to using the NPV rule for a single investment ( ) and the loss due to using the NPV rule for multiple investment projects ( ). We also plot the loss due to using the deterministic model for a single investment ( ) and the loss due to the deterministic model for two sequential investment projects ( ).

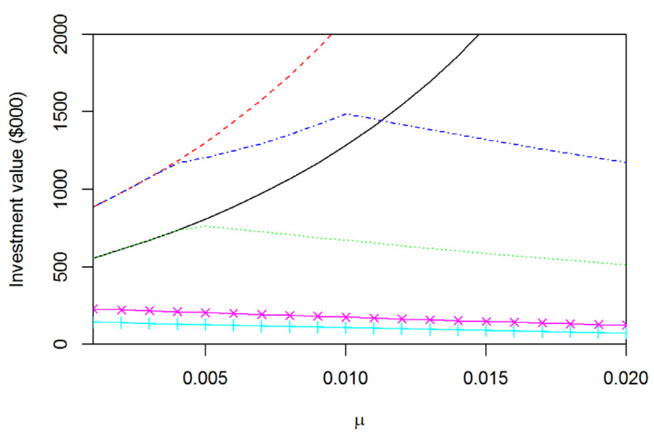

Fig. 7. Impact of different values for the growth rate of the Poisson intensity ($0 < \mu < 0.02$) as a proxy for different climate change scenarios on option values and the loss due to using the NPV rule. We plot the investment option value for alternative projects ( ), the option value of sequential investment ( ), the loss due to using the NPV rule for a single investment ( ) and the loss due to using the NPV rule for multiple investment projects ( ). We also plot the loss due to using the deterministic model for a single investment ( ) and the loss due to the deterministic model for two sequential investment projects ( ).

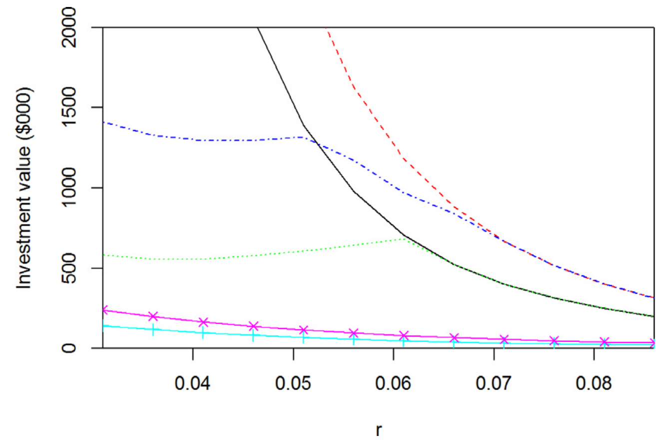

Fig. 8. Impact of different values of the discount rate ($3\% < r < 9\%$) on option values and the loss due to using an NPV rule. We plot the investment option value for alternative projects ( ), the option value of sequential investment ( ), the loss due to using the NPV rule for a single investment ( ) and the loss due to using the NPV rule for multiple investment projects ( ). We also plot the loss due to using the deterministic model for a single investment ( ) and the loss due to the deterministic model for two sequential investment projects ( ).

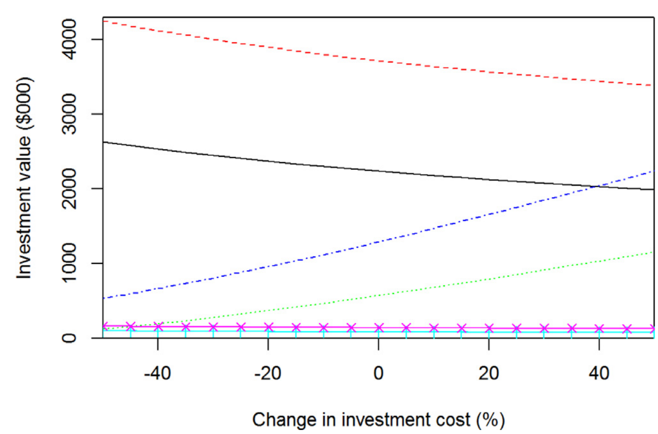

Fig. 9. Impact of reduction and increase in the the investment costs ($-40\% < \Delta I_M < 40\%$) on option values and the loss due to using NPV rule. We plot the investment option value for alternative projects ( ), the option value of sequential investment ( ), the loss due to using the NPV rule for a single investment ( ) and the loss due to using the NPV rule for multiple investment projects ( ). We also plot the loss due to using the deterministic model for a single investment ( ) and the loss due to the deterministic model for two sequential investment projects ( ).

by more than 10%. It is therefore more important to use the real options model for large projects rather than small projects.

The impact of uncertainty is further examined by letting the uncertainty parameter σ vary from 0.15 to 0.45. As shown in Fig. 6 , as the uncertainty increases, the investment option values as well as the losses due to suboptimal investment rule increases, but at decreasing rates. The value of the option to invest in a sequence of projects is more sensitive to the uncertainty parameter, and so are the losses associated with the sequential investment option values. This implies that for practical contexts such as climate change adaptation, where the uncertainty is large, real options models should be used to guide investment decisions, especially for sequential project investment.

The impact of a more serious climate change scenario is further explored by allowing μ to vary over a range from 0 to 2% (Fig. 7). Under a more serious climate change scenario, the value of investment options are higher since adaptation projects are more beneficial. Losses due to the NPV rule are equal to the option values, which increases with μ when climate change is moderate. However, when climate change is sufficiently serious, leading to positive NPVs for immediate investment into the projects, the losses decrease as μ becomes larger. In the latter case, the decreases in the losses are due to the fact that further increases in μ raise the value of immediate investment more than the option value. In contrast, the losses by the deterministic model slightly decreases under more serious climate change scenario.

We further investigate the impact of the applied discount rate by allowing the discount rate to vary from 3 to 9% (Fig. 8). The discount rate has a large impact on the option values as well as the losses due to suboptimal investment. It is apparent from Fig. 8 that as the discount rate becomes lower, future cash flows become more important, increasing the benefits of using a sophisticated investment model.

The impact of investment cost is further examined by allowing the change in investment costs to vary from −40 to 40% (Fig. 9). Increases in investment costs have much lower impact on the option values compared to the value obtained by immediate investment.

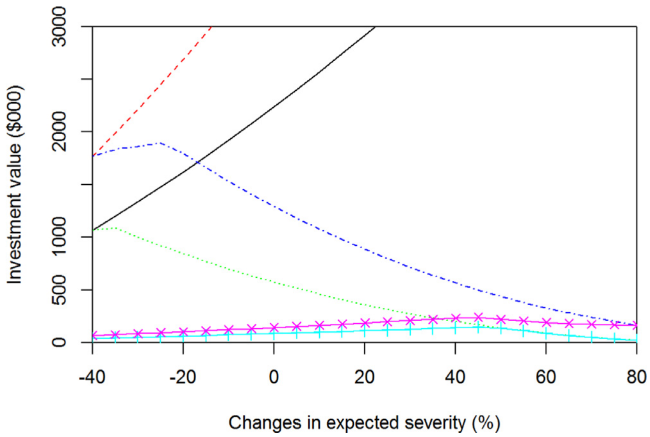

Fig. 10. Impact of reduction and increase in the expected loss severity ($-40\% < \Delta\beta < 80\%$) on option values and the loss due to using NPV rule. We plot the investment option value for alternative projects (———), the option value of sequential investment (– – – –), the loss due to using the NPV rule for a single investment (.........) and the loss due to using the NPV rule for multiple investment projects (–.–.–.). We also plot the loss due to using the deterministic model for a single investment (——+) and the loss due to the deterministic model for two sequential investment projects (——×).

This is because optimal investment will occur in the future, and after discounting, any rise in investment costs will have lower impacts. This explains also the high increase in the loss for the NPV investment rule in comparison to the losses by the deterministic model.

Finally we examine the impact of the expected loss severity, by allowing the change in the expected value of loss severity to vary from −40 to 80% (Fig. 10). As the expected loss severity increases, the NPVs of the projects and the values of the investment options increase. For the range of expected loss severity, where the NPV rule does not result in immediate investment, the loss due to applying a suboptimal investment rule increases. However, when the expected loss severity is sufficiently large to recommend immediate investment based on a NPV rule, further increases in the expected loss severity will reduce the losses due to the use of a suboptimal rules. This is because higher expected loss severity makes it optimal to invest earlier and makes immediate investment less suboptimal. Losses resulting from the application of a deterministic model instead of a real options model are of much smaller magnitude, but also indicate an initially increasing, and then decreasing relationship with the expected loss severity.

3.4. Incorporation of co-benefits

Incorporation of the annual co-benefits offered by investment projects into the modelling framework is straightforward since annual co-benefits are similar to maintenance costs. From the investment perspective, a project that has an annual co-benefit of $40,000 and a maintenance cost of $100,000 is equivalent to a project that has no co-benefit and a maintenance cost of $60,000. The main challenge is rather the quantification of co-benefits. For example, to incorporate the co-benefits of the fire trails, the recreational benefits of using the fire trails for hiking and/or biking would need to be quantified.

The literature on the estimation of the recreational benefits of hiking or biking trails in parks is scarce. Some studies have been conducted for the U.S. (Lee, Kim, Graefe, & Chi, 2013; Scarpa, Thiene, & Hensher, 2010) and Austria (Koemle & Morawetz, 2016), but no study has been done for Australia. Due to the differences in local characteristics of parks and in the income and preferences of amenity users, we cannot use the results from other countries for Australia.

Non-market values of co-benefits can be quantified using revealed preference methods (travel cost, hedonic pricing) or stated preference methods (contingent valuation, choice modelling) (Mendelsohn and Olmstead, 2009). The travel cost method estimates the recreational value of amenities such as hiking tracks based on the costs that people incur to travel to the hiking site while hedonic pricing methods use the market price of related goods such as house prices to infer the value of the amenity. These methods rely on historical data on travel costs or house prices. In contrast, contingent valuation and choice modelling rely on surveys that explore how the willingness to pay of amenity users changes when the supply of the amenity is altered. Estimation of co-benefits, therefore, requires the development of a reasonably comprehensive data set or to conduct a survey, which is beyond the scope of this paper.

Without reliable estimates of the co-benefits, only a qualitative assessment of these benefits on the investment decision can be examined. Incorporating co-benefits effectively reduces the maintenance cost and therefore reduces the effective investment cost of the project, as discussed in Section 2.4. Consequently, the NPV of the project would be increased and the investment delay would be reduced. Properly accounting for co-benefits, therefore, will typically result in earlier investment in adaptation projects. It is also important to note that if the co-benefits of one or several projects are significant, these might possibly change the order of sequential investment.

4. Implications, contributions and limitations

In this paper, we provide a new modelling framework that allows to analyze and select optimal investment in catastrophic risk reduction projects under uncertainty. A valuable feature of our model is that it allows to incorporate the uncertainty of climate change predictions into the analysis of investment decisions. The framework also allows to analyze alternative investments where the decision maker can select only one project among many, possibly due to budget constraints. Importantly, it further allows to select optimal sequential investments where the decision maker is free to choose any investment projects that improve the social welfare.

We illustrate the application of the framework in a case study on bushfire risk management for a local government area in Australia. Our results suggest that the value of investment flexibility under uncertainty is large relative to the NPV of the considered projects. Using a simple NPV decision rule would result in substantial losses, in comparison to the application of a real options framework. The loss is higher for the case where the decision maker is not constrained to select one investment project only, but can invest in several projects at the same time or in a sequential setting. The losses can be reduced by using a deterministic model that allows to defer investment until catastrophic risk grows to higher levels. The deterministic model, however, ignores the uncertainty in catastrophic frequency such that using a real options models can further increase the value of the investment.

An interesting finding from the empirical application is that if the decision maker faces budget constraints and can invest in one project only, it is optimal to select the largest feasible project in order to obtain the largest reduction in catastrophic risk. On the other hand, if the decision maker is free to select investment projects to maximize the long term social welfare, it is typically best to select a low sunk cost project first to preserve the investment flexibility and only invest in a high sunk cost project when the impact of climatic change is sufficiently large. Compared to the case of a single project investment, investment starts earlier when

a sequence of projects is considered. Furthermore, the optimal investment strategy for sequential investment is consistent with the results obtained by a more qualitative analysis in previous studies, see e.g., Hallegatte (2009), where it is suggested to invest in low sunk cost projects first to preserve flexibility. Sequential investment analysis is therefore essential for climate change adaptation projects. These results also suggest that in order to enable effective adaptation, financial assistance from higher government levels to local governments may be necessary, so that local governments are less constrained financially and are able to preserve investment flexibility. Our findings illustrate that in particular for managing sequential investments, local governments should at least base their investment decisions on a deterministic dynamic investment model instead of using a simple NPV rule. Therefore, financial support to local governments should be provided in combination with additional support towards understanding the optimal timing and selection of adaptation projects.

The conducted sensitivity analysis indicates that optimal investment decisions, including which project to select for alternative investment as well as the order of the invested projects in a sequential investment framework, are relatively robust to changes in the considered key parameters. Further, the option values of the projects are relatively insensitive to investment costs, while they are quite sensitive to the impact of climatic change, the applied discount rate, and the expected loss severity. The loss due to the use of a NPV rule instead of a real options model is highly sensitive to investment cost and the expected loss severity. We further find that the loss resulting from the application of a deterministic dynamic model, instead of a real options approach, is highly sensitive to uncertainty, the discount rate and the expected severity. This implies that for large scale projects, it is important to at least base the investment decision on a deterministic dynamic model. When both uncertainty and the scale of the project are large, it is important to use a real options approach.

We also discuss how co-benefits of adaptation projects can be incorporated in our model. Incorporating co-benefits will reduce the effective investment cost of the project and lead to earlier investment. However, the current literature on quantifying co-benefits of bushfire adaptation projects is scarce and future studies in this area would help to reduce delay in climate change adaptation.

There are also several limitations to this study that need to be acknowledged. We adopt a simplifying assumption that loss severity is independent from loss frequency. In reality, loss frequency may be correlated with loss severity via fuel stocks and other variables.[9] A severe fire may significantly reduce the fuel stocks and defer the next fire to a distant future, while a longer time period without a fire may accumulate fuel and possibly increase the likelihood for the next bushfire to be more severe. This interaction might possibly reduce the incentive to wait for the fire frequency to reach a high level before investing and therefore lower the investment threshold. This interaction is interesting and one way to model it is by extending our framework to allow loss frequency and loss severity to depend on factors. This could involve common factors that impact on both loss frequency and loss severity as well as factors that affect loss frequency or loss severity only. Factors may be observable when data on climate variables and fuel stock are available, but they can be also modelled as latent variables. In addition, factors that represent climate variables may be modelled to follow a random walk while factors that represent fuel stocks are more likely to be mean-reverting. When the fuel stocks are high, there is a tendency for bushfires to occur, which in return will reduce fuel stocks. On the other hand, when the fuel stocks are low, fire is less likely to occur, allowing the fuel stocks to build up. Models with latent factors are often known as dynamic factor models and have been extensively used in systemic risk studies, see e.g., Koopman and Lucas (2008). Conditional on factors, loss severity can be assumed to be independent from loss frequency. The development of such a model for bushfire is beyond the scope of this paper and will be left to a future study.

[9] We would like to thank one of the referees for raising this point.

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